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Technical Brief

Estimation of Disk Burst Pressures Using Limit Analysis Collapse Loads

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Introduction

Jones and Holliday [1] recently reported simulations of clamped circular disks employing a continuum finite element method and elastic/plastic material models. An interesting conclusion of that work is that using a small strain small deflection elastic perfectly plastic model (SSSD EPP in the notation of [1]) is a conservative way to estimate burst pressures. Limit analysis-

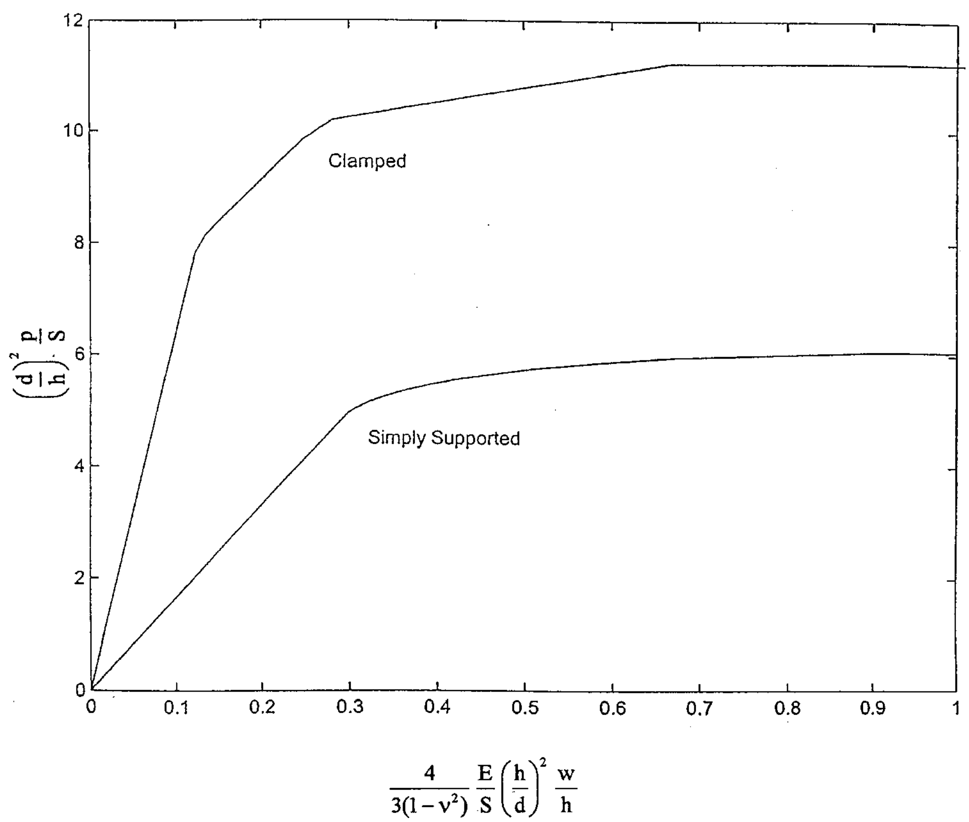

Fig. 1 Dimensionless load central deflection curves

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Table 1 Numerical factors for Eq. (1)

Yield Criterion	Tresca [2]	Johansen [3]	Von Mises [4]
N	11.26	12.0	12.5

Table 2 Collapse loads

Material	S (MPa)	p_c (MPa)		
		N = 11.26	N=12.0	N=12.5
ABC-C	221	0.969	1.033	1.076
A-533-B	331	1.452	1.547	1.612
304 SS	234	1.026	1.094	1.139

based closed-form expressions for collapse loads of clamped uniformly loaded circular plates exist in the literature. The purpose of the present note is to compare these expressions to the limit load results obtained numerically in [1].

Limit Analysis

Limit analysis of axisymmetrically loaded circular plates is based on the idealization of so-called "full section" yielding in which cross sections are assumed to yield as a whole. This yielding mechanism would be exact for an idealized sandwich plate consisting of two infinitesimally thin membranes carrying normal stresses separated by a core carrying only transverse shear stresses. It is, however, approximate for a plate of uniform properties in which yielding starts at the top and bottom and proceeds inward.

The full section yielding concept has been employed in connection with small strain small deflection rigid perfectly plastic plate models (SSSD RPP) to compute the collapse loads of uniformly loaded circular plates by Hopkins and Prager [2] using the Tresca yield criterion, by Wood [3] using the Johansen yield criterion, and by Hopkins and Wang [4] using the Von Mises yield criterion. All results can be put in the common form

$$p_c = NS(h/d)^2 \quad (1)$$

where N is a numerical factor dependent on the yield criterion listed in Table 1, S is interpreted as the yield stress, h is the plate thickness, and d is the plate diameter.

Limit analysis is most easily performed using SSSD RPP plate models, but does not produce complete load/deflection behavior prior to collapse. An alternative is to carry out the limit analysis based on SSSD EPP plate models as has been done by Tekinalp [5] for $\upsilon=0.5$ and by Wu [6] for $0 \leqslant \upsilon \leqslant 0.5$ (υ being Poisson's ratio) using the Tresca yield criterion. This work reproduces Eq. (1) with $N=11.26$ and provides load/deflection information. A typical dimensionless pressure/central deflection curve for a 304 stainless steel ($\upsilon=0.28$) plate is shown in Fig. 1. For comparison, the corresponding pressure/central deflection curve for a simply supported 304 stainless steel plate is also shown. In Fig. 1, p is the pressure, w is the central deflection, and E is the modulus of elasticity.

Results and Discussion

The limit load predictions of [1] are based on the limit load strength parameter defined therein. For consistency, the quantity S appearing in Eq. (1) must be reinterpreted as being this limit load strength parameter. Table 2 shows the values of S reported in [1] for the three steels considered therein together with collapse pressures predicted by Eq. (1) for the three yield criteria. These numbers vary from 70–82% of the corresponding limit pressures reported in Table 6 of [1]. It would thus appear that Eq. (1) is a viable working equation for obtaining conservative estimates of circular disk burst pressures.

References

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